In a nutshell: Muller's method

Given a continuous real-valued function f of a real variable with three initial approximations of a root x_{-2}, x_{-1} and x_0 where $|f(x_{-2})| \geq |f(x_{-1})| \geq |f(x_0)| > 0$, rearranging them if necessary. If one is zero, we have already found a root. This algorithm uses iteration, quadratic interpolation and a solution to the quadratic equation to approximate a root.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the root cannot exceed this value.
ε_{abs}	The value of the function at the approximation of the root cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. If $|f(x_{-2})| \geq |f(x_{-1})| \geq |f(x_0)| > 0$ does not hold, rearrange the values so as to ensure this is true.
4. The next approximation to the root will be x_k plus the smaller root of the quadratic polynomial that interpolates the points $(x_{k-2} - x_k, f(x_{k-2}))$, $(x_{k-1} - x_k, f(x_{k-1}))$ and $(0, f(x_k))$.

   Let
   $$a \leftarrow \frac{\left(f(x_{k-1}) - f(x_{k-2})\right)x_k + \left(f(x_{k-2}) - f(x_k)\right)x_{k-1} + \left(f(x_k) - f(x_{k-1})\right)x_{k-2}}{(x_{k-1} - x_{k-2})(x_{k-2} - x_k)(x_k - x_{k-1})}$$
   $$b \leftarrow \frac{\left(f(x_{k-1}) - f(x_{k-2})\right)x_k^2 + 2\left(\left(f(x_{k-2}) - f(x_k)\right)x_{k-1} + x_{k-2}\left(f(x_k) - f(x_{k-1})\right)\right)x_k - \left(f(x_{k-2}) - f(x_k)\right)x_{k-1}^2 - \left(f(x_k) - f(x_{k-1})\right)x_{k-2}^2}{(x_{k-1} - x_{k-2})(x_{k-2} - x_k)(x_k - x_{k-1})}.$$
   $$c \leftarrow f(x_k)$$

   If $a = 0$, the three points are in a straight line, so let $x_{k+1} \leftarrow x_k - \frac{c}{b}$;

   otherwise, if $b^2 - 4ac < 0$, the roots are imaginary, so return signalling a failure to converge;

   otherwise, if $b > 0$, let $x_{k+1} \leftarrow x_k - \frac{2c}{b + \sqrt{b^2 - 4ac}}$,

   if $b < 0$, let $x_{k+1} \leftarrow x_k - \frac{2c}{b - \sqrt{b^2 - 4ac}}$,

   otherwise it must be that $b = 0$, so let $x_{k+1} \leftarrow x_k - \sqrt{-\frac{c}{a}}$.

   a. If x_{k+1} is not a finite floating-point number, so return signalling a failure to converge.
   b. If $|x_{k+1} - x_k| < \varepsilon_{\text{step}}$ and $|f(x_{k+1})| < \varepsilon_{\text{abs}}$, return x_{k+1}.
5. Increment k and return to Step 2.

Convergence

If h is the error, it can be show that the error decreases according to $O(h^{\mu})$ where $\mu \approx 1.8393$ is the real root of $x^3 - x^2 - x - 1 = 0$.